**Linear independence**

* **rank**: counts **the number** of genuinely independent rows in the matrix A.
* **linearly independent**: *c1v1 + … + ckvk = 0* only happens when *c1 = … = ck = 0* 
  + The columns of **A** are independent exactly when ***N****(A) = {****zero vector****}*
  + **The r nonzero rows** of an echelon matrix U and a reduced matrix R arelinearly independent.
  + **The r columns** that contain pivots are linearly independent.
* **linearly dependent**: if any c’s are nonzero, the v’s are linearly dependent.
  + One vector is a combination of the others.
  + **A set of n vectors** in Rm must be linearly dependent if n>m

**spanning a subspace**

* for **a set of vectors** to span a space:
  + **Their combinations** produce the whole space.
  + if a **vector space *V*** consists of all linear combinations of ***w***1, …, ***wl***, then ***w***1, …, ***wl*** span the space ***V***. every vector***v*** in ***V*** is some combination of the ***w***’s.

**Basis for a vector space**

* **spanning** involves the **column space**:
  + to decide if **b** is a combination of the columns, solve Ax=b
* **independence** involves the **nullspace**.
  + to decide if the columns are **independent**, solve Ax=0
* **definition**
  + **A basis** for ***V*** is **a sequence of vectors** having two properties at once.
    - 1, **the vectors** are linearly independent (not too many vectors)
    - 2, **they** span the space ***V*** (not too few vectors)
* **properties of basis:**
  + there is one and only one way to write ***v*** as a combination of the basis vectors.
  + **a vector space** doesn’t have a unique basis.

**dimension for a vector space**

* **the dimension of a space** is **the number of vectors** in every basis.
  + **Any two bases** for a vector space ***V*** contain **the same** number of vectors(**dimension**)
  + **The dimension** of the column space equals **the rank** of the matrix.
  + the dimension of the empty set is zero.
* any **linearly independent set** in ***V*** can be extended to a basis, by adding more vectors if necessary.
  + a basis is **a maximal independent set**. It cannot be made **larger** without losing independence.
* any **spanning set** in ***V*** can be reduced to a basis, by discarding vectors if necessary.
  + a basis is also **a minimal spanning set**. It cannot be made **smaller** and still span the space.